Exercise 8

Verify the given linear approximation at $a = 0$. Then determine the values of x for which the linear approximation is accurate to within 0.1.

$$(1+x)^{-3} \approx 1 - 3x$$

Solution

Plugging in $x = 0$ to the function yields $(1+x)^{-3} = 1$, so $(0, 1)$ is the point on the curve that the tangent line goes through. Taking the derivative of the function yields

$$\frac{d}{dx}(1+x)^{-3} = -3(1+x)^{-4} \cdot \frac{d}{dx}(1+x) = -3(1+x)^{-4} \cdot 1 = -3(1+x)^{-4}.$$

Set $x = 0$ to get the slope of the tangent line.

$$\left.\frac{d}{dx}(1+x)^{-3}\right|_{x=0} = -3(1+0)^{-4} = -3$$

Use the point-slope formula to get the equation of this line.

$$y - 1 = -3(x - 0)$$

$$y - 1 = -3x$$

$$y = 1 - 3x$$

As a result, the linearization to $(1+x)^{-3}$ at 0 is

$$L(x) = 1 - 3x.$$

Now find the values of x for which the linear approximation is accurate to within 0.1.

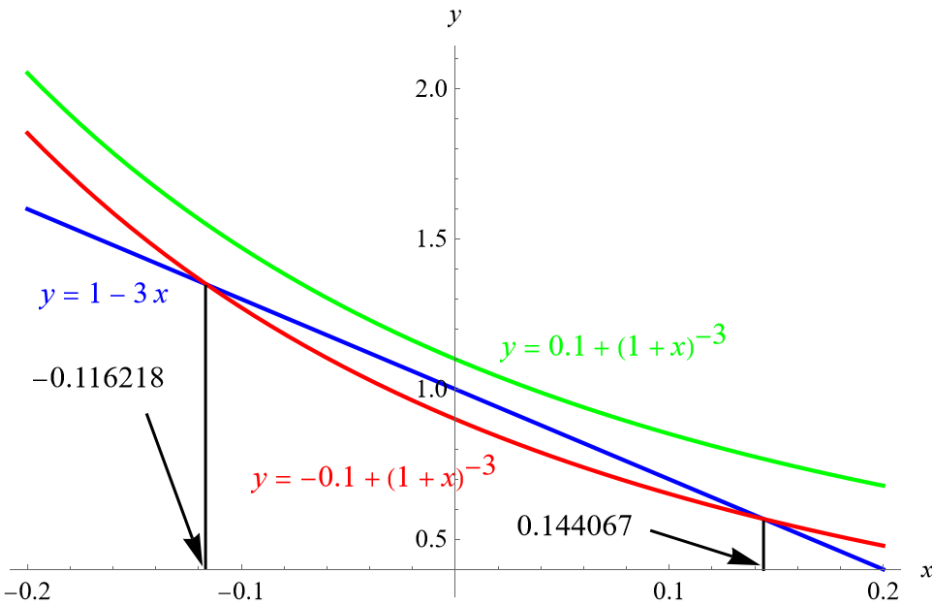
$$|(1+x)^{-3} - (1-3x)| < 0.1$$

$$|(1-3x) - (1+x)^{-3}| < 0.1$$

$$-0.1 < (1-3x) - (1+x)^{-3} < 0.1$$

$$-0.1 + (1+x)^{-3} < 1 - 3x < 0.1 + (1+x)^{-3}$$

Plot each of these functions versus x.



The linear approximation stays between the curves for

$$-0.116218 < x < 0.144067.$$

This is the interval that the linear approximation is accurate to within 0.1.